Math 474 - Homework # 8
Continuous Distributions

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1. Suppose that you know that X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} 0 & \text{if } x \leq 10 \\ \dfrac{10}{x^2} & \text{if } x > 10 \end{cases}$$

   (a) Draw a picture of f.

   (b) Show that indeed f is a probability density function.

   (c) Calculate $P(1 \leq X \leq 5)$

   (d) Calculate $P(-1 \leq X \leq 30)$

   (e) Calculate $P(X > 20)$

   (f) Find the cumulative distribution function $F(t)$ of X.

   (g) Draw a picture of F.

   (h) Calculate $E[X]$

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2. Suppose that you know that X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} c(1 - x^2) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   where c is some real number.

   (a) What does c have to be to make sure that f is a probability density function?

   (b) Draw a picture of f.

   (c) Calculate $P(X < 0)$

(d) Calculate $P(-1 \le X < 1/2)$

(e) Calculate $P(-10 \le X < 1/2)$

(f) What is the cumulative distribution function $F(t)$ of X ?

(g) Draw a picture of F.

(h) Calculate $E[X]$

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3. Let $\lambda > 0$. Consider the exponential probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that f is indeed a probability density function.

   (b) Calculate $E[X]$

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4. Suppose that you know that X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} a + bx^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   If $E[X] = \dfrac{3}{5}$, find a and b.

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5. You arrive at a bus stop at 10:00, knowing that the bus will arrive at some time uniformly distributed between 10:00 and 10:30.

   (a) What is the probability that the bus will arrive between 10:05 and 10:11?

   (b) What is the probability that you will have to wait longer than 10 minutes?

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6. The time (in hours) required to repair a machine is an exponential random variable with parameter $\lambda = \dfrac{1}{2}$.

   (a) Find the probability that it takes between 0 - 1 hour to repair.

   (b) Find the probability that a repair time exceeds 2 hours.
